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FUZZY SUBGROUPS COMPUTATION OF FINITE GROUP BY USING THEIR LATTICES

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Abstract: This paper gives a method to determine the number of fuzzy subgroups of finite group G using diagram of lattice subgroups of G . This method can be used for abelian and non-abelian groups. First, an equivalence relation on the set of all fuzzy subgroups of group G is defined. Second, this paper derive some theorems for determination of the number of fuzzy subgroups associated with some chains on the lattice subgroups of G . Then the demonstration of the method is given by determining the number of fuzzy subgroups of some finite group G .

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Key Words: equivalence, lattice, chain, fuzzy subgroup

1. Introduction

The concept of fuzzy sets was first introduced by Zadeh in 1965 (see [1]). The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups by Rosenfeld in 1971 (see [2]). Since the first paper by Rosenfeld, researchers have sought to characterize the fuzzy subgroups

of various groups, particularly of finite abelian groups. Several papers have studied the particular case of finite cyclic group. Laszlo (see [3]) studied the construction of fuzzy subgroup of groups of the orders one to six. Zhang and Zou (see [4]) have determined the number of fuzzy subgroups of cyclic groups of the order p^n where p is a prime number. Murali and Makamba (see [5], [6]), considering a similar problem, found the number of fuzzy subgroups of abelian groups of the order $p^n q^m$ where p and q are different primes, while Tarnauceanu and Bentea (see [7]) have found this number for finite abelian groups.

In this paper, we take a different approach to determine the number of fuzzy subgroups of some finite groups G . We determine this number by observing the diagram of lattice subgroups of G . First, we define the equivalence relation on the set of all fuzzy subgroups of group G (see Definition 3). This definition differs from that of Murali and Makamba (see [5], [6], [8]), but is equivalent to the definitions of Dixit (see [9]), Zhang (see [4]) and Tarnauceanu (see [7], [10]). We prefer to use this definition because it generalizes the definition used by Murali. We observe patterns of diagram of lattice subgroups of G and determined its influence on the formula of the number of fuzzy subgroups. This method can be used not only for abelian group, but also for non-abelian as well.

We found some formulas in more general. Results of Zhang (see theorem 3.1.(1)) in [4] and Murali (see proposition 3.3 and theorem 3) in [5] can be viewed as special cases from our formula (see Theorem 18, Example 17 and Corollary 23, respectively). Tarnauceanu (see [7]) asserted that computing the number of fuzzy subgroups of $Z_3^2 = Z_2 \times Z_2 \times Z_2$ by using "the method of direct calculation" is too difficult. We will show, however, that it is easy to compute using our method (see Example 15).

2. Preliminaries

In this paper, a group G is assumed to be a finite group. First of all, we present some basic notions and results that will be used later.

Definition 1. Let X be a nonempty set. A fuzzy subset of X is a function from X into $[0, 1]$.

Definition 2. (Rosenfeld, see [2]). Let G be a group. A fuzzy subset μ of G is called a fuzzy subgroup of G if:

- (1) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in G,$
- (2) $\mu(x^{-1}) \geq \mu(x), \forall x \in G.$

If μ is a fuzzy subgroup of group G and $\mu(G) = \{\theta_1, \theta_2, \dots, \theta_n\}$, then we

assume that $\theta_1 > \theta_2 > \dots > \theta_n$.

15 Theorem 3. (Rosenfeld, see [2]). Let e denote the identity element of G . If μ is a fuzzy subgroup of G , then:

$$(1) \mu(e) \geq \mu(x), \forall x \in G,$$

$$(2) \mu(x^{-1}) = \mu(x), \forall x \in G.$$

Theorem 4. (Sulaiman and Abdul Ghafur, see [11]). A fuzzy subset μ of G is a fuzzy subgroup of G if and only if there is a chain of subgroups G , $P_1(\mu) \leq P_2(\mu) \leq P_3(\mu) \leq \dots \leq P_n(\mu) = G$ such that μ is in the form:

$$\mu(x) = \begin{cases} \theta_1, & x \in P_1(\mu) \\ \theta_2, & x \in P_2(\mu) \\ \vdots & \\ \theta_m, & x \in P_m(\mu) \end{cases} \quad (1)$$

If there is no confusion, then $P_i(\mu)$ as in (1) can simply be written as P_i . We define *length* (or *order*) of fuzzy subgroup μ in (1) as be m .

Example 5. Consider the group $G = Z_{12}$. Define functions μ, γ, α and β as follows:

$$\begin{aligned} \mu(x) &= \begin{cases} 1, & x \in \{0, 2, 4, 6, 8, 10\} \\ \frac{1}{2}, & x \in \{1, 3, 5, 7, 9, 11\} \end{cases}, \\ \gamma(x) &= \begin{cases} 1, & x \in \{0, 1, 2, 3, 4, 5\} \\ \frac{1}{2}, & x \in \{6, 7, 8, 9, 10, 11\} \end{cases}, \\ \alpha(x) &= \begin{cases} 1, & x \in \{0, 4, 8\} \\ \frac{1}{2}, & x \in \{2, 6, 10\} \\ \frac{1}{3}, & x \in \{1, 3, 5, 7, 9, 11\} \end{cases}, \\ \beta(x) &= \begin{cases} \frac{1}{2}, & x \in \{0, 4, 8\} \\ \frac{1}{4}, & x \in \{2, 6, 10\} \\ 0, & x \in \{1, 3, 5, 7, 9, 11\} \end{cases}. \end{aligned}$$

Note that

$$P_1(\mu) = \{0, 2, 4, 6, 8, 10\}, P_2(\mu) = \{0, 2, 4, 6, 8, 10\} \cup \{1, 3, 5, 7, 9, 11\} = Z_{12}.$$

Thus $P_1(\mu)$ and $P_2(\mu)$ both are subgroups of Z_{12} . By Theorem 2, μ is fuzzy subgroup of Z_{12} . Similarly, we can show that α and β both are fuzzy subgroups of Z_{12} . Note that $P_1(\gamma) = \{0, 1, 2, 3, 4, 5\}$ is not a subgroup of Z_{12} , so γ is not a fuzzy subgroup of Z_{12} .

3. Equivalence Relation on Fuzzy Subgroups

Without any equivalence relation on fuzzy subgroups of group G , the number of fuzzy subgroups is infinite, even for the trivial group $\{e\}$. So we define equivalence relation on the set of all fuzzy subgroups of a given group. Using this equivalent, we compute the number of distinct equivalence classes of fuzzy subgroups of group G .

Definition 6. Let μ, γ be fuzzy subgroups of G of the form

$$\mu(x) = \begin{cases} \theta_1, & x \in P_1 \\ \theta_2, & x \in P_2 \setminus P_1 \\ \theta_3, & x \in P_3 \setminus P_2 \\ \dots & \\ \theta_n, & x \in P_n \setminus P_{n-1} \end{cases}, \gamma(x) = \begin{cases} \delta_1, & x \in P_1 \\ \delta_2, & x \in P_2 \setminus P_1 \\ \delta_3, & x \in P_3 \setminus P_2 \\ \dots & \\ \delta_m, & x \in P_m \setminus P_{m-1} \end{cases}.$$

Then we define that μ and γ are *equivalent* if:

- (1) $m = n$, and
- (2) $P_i = M_i, \forall i \in \{1, 2, 3, \dots, n\}$.

We write $\mu \sim \gamma$. It is easily checked that this relation is indeed an equivalence relation. Two fuzzy subgroups of G are said to be different if they are not equivalent. In this paper, the number of fuzzy subgroups of group G means the number of distinct equivalence classes of fuzzy subgroups.

Example 7. Let μ, α and β be fuzzy subgroups as in Example 5. Since $|\mu| \neq |\alpha|$ and $|\mu| \neq |\beta|$, by Definition 6 we determine that μ and α are not equivalent. Fuzzy subgroups μ and β are not equivalent either. Note that $|\alpha| = |\beta|$ and it is easy to show that $P_i(\alpha) = P_i(\beta), \forall i \in \{1, 2, 3\}$. Thus, $\alpha \sim \beta$.

4. Method of Counting Using Lattices

In this section we discuss a method to determine the number of fuzzy subgroups of a group G using a diagram of lattice subgroups of G . We observe at every chain and pattern of diagram of lattice subgroups of G to compute the number of fuzzy subgroup. This method can be used both for abelian and non-abelian groups. We denote the number of fuzzy subgroups of G as $n(F_G)$, while the number of fuzzy subgroups μ of G with $P_1(\mu) = H$ is denoted by $n(F_{P_1=H})$. From Theorem 4, we have

$$n(F_G) = \sum_{H \leq G} n(F_{P_1=H}) \quad (2)$$

Theorem 8. Let $G_1 < G_2 < \dots < G_{k-1} < G_k = G$ be the only maximal chain from G_1 to G on the lattice subgroups of G . Then:

$$n(F_{P_1=G_1}) = \begin{cases} 1, & k=1,2 \\ 2^{k-2}, & k > 2 \end{cases}.$$

Proof. If $k=1$, then $G_1 = G$. We have only one fuzzy subgroup of G where $P_1 = G$. Its length is 1, that is, $\mu(x) = \theta_1, \forall x \in G$. If $k=2$, we have only one fuzzy subgroup of G where $P_1 = G_1$. Its length is 2, that is,

$$\mu(x) = \begin{cases} \theta_1, & x \in G_1 \\ \theta_2, & x \in G \setminus G_1 \end{cases}.$$

If $k > 2$, the part of diagram of lattice subgroups of G is $G_1 - G_2 - \dots - G_k = G$. Let μ be the fuzzy subgroup of G where $P_1(\mu) = G_1$. Since $G_1 \neq G$ and since the maximal chain $G_1 \subset G_2 \subset \dots \subset G_k = G$ contains k subgroups of G , then the possible length of μ is 2, 4, ..., $(k-1)$ or k . Let n be an arbitrary element of $\{2, 3, 4, \dots, k\}$. We will count the number of fuzzy subgroups μ of length n where $P_1 = G_1$ and $P_n = G$. For $P_2, P_3, \dots, P_n$, we can choose one of $(k-2)$ pieces of the existing subgroup $G_2, G_3, \dots, G_{k-1}$. So the number of fuzzy subgroups of G with $P_1 = G_1$ of length n is C_{n-2}^{k-2} . Thus, the number of fuzzy subgroups of G with $P_1 = G_1$ is equal to $C_0^{k-2} + C_1^{k-2} + C_2^{k-2} + \dots + C_{k-3}^{k-2} + C_{k-2}^{k-2} = 2^{k-2}$. $\square$

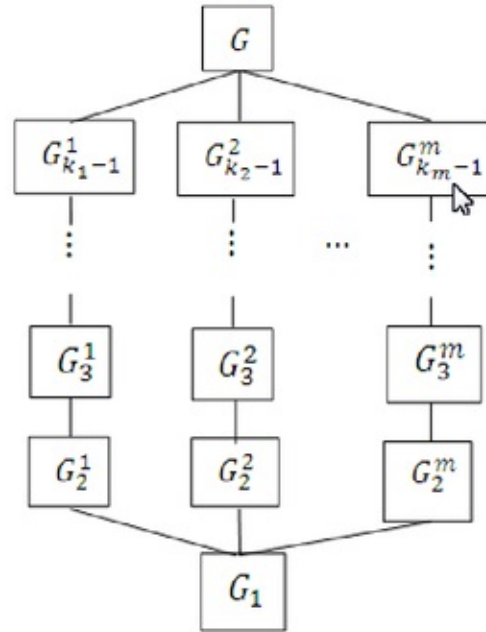
Corollary 9. Let $G_1 < G_2 < \dots < G_{k-1} < G_k = G, k \geq 2$ be the only maximal chain from G_1 to G on the lattice subgroups of G . For $m, 1 \leq m \leq k-1$ we have $n(F_{P_1=G_m}) = 2^{k-m-1}$.

Proof. Use Theorem 8. $\square$

Corollary 10. Let $G_1 < G_2 < \dots < G_{k-1} < G_k = G, k \geq 3$ be the only maximal chain from G_1 to G on the lattice subgroups of G . For $m, 1 \leq m \leq k-2$ we have $n(F_{P_1=G_m}) = 2 \cdot n(F_{P_1=G_{m+1}})$.

Proof. Using Theorem 3, we have $n(F_{P_1=G_m}) = C_0^{k-m-1} + C_1^{k-m-1} + C_2^{k-m-1} + \dots + C_{k-m-1}^{k-m-1}$ and $n(F_{P_1=G_{m+1}}) = C_0^{k-m-2} + C_1^{k-m-2} + C_2^{k-m-2} + \dots + C_{k-m-2}^{k-m-2}$. Then using Pascal's recursion formula (see [12]), $C_k^n = C_{k-1}^{n-1} + C_k^{n-1}$, we obtain $n(F_{P_1=G_m}) = C_0^{k-m-1} + C_0^{k-m-2} + C_1^{k-m-2} + C_1^{k-m-3} + C_2^{k-m-3} + C_2^{k-m-4} + \dots + C_{k-m-3}^{k-m-2} + C_{k-m-2}^{k-m-2} + C_{k-m-1}^{k-m-1} = 2[n(F_{P_1=G_{m+1}})]$. $\square$

Theorem 11. Let H be a subgroup of G , and let the set of all subgroups of G which contain H (but are not equal to H) be $\{H_1, H_2, H_3, \dots, H_k\}$. Then $n(F_{P_1=H}) = \sum_{i=1}^k n(F_{P_1=H_i})$.

Figure 1: Part of the diagram of lattice subgroups G .

Proof. We have $n(F_{P_1=H}) = \sum_{i=1}^k n(F_{P_1=H; P_2=H_i})$. Note that $\forall i \in \{1, 2, \dots, k\}$,

$$n(F_{P_1=H; P_2=H_i}) = n(F_{P_1=H_i}).$$

Hence, we obtain $n(F_{P_1=H}) = \sum_{i=1}^k n(F_{P_1=H_i})$. $\square$

Theorem 12. Let G_1 be a subgroup of G . If there are m ($m \geq 2$) maximal chains from G_1 to G on the lattice subgroup G of length $k_1, k_2, \dots, k_m$, and every pair of those chains is disjoint except in G_1 and G , then $n(F_{P_1=G_1}) = \sum_{i=1}^m 2^{k_i-2} - (m-1)$.

Proof. Let the chains be

$$\begin{aligned} G_1 &\subset G_2^1 \subset G_3^1 \subset \dots \subset G_{k_1-1}^1 \subset G, \\ G_1 &\subset G_2^2 \subset G_3^2 \subset \dots \subset G_{k_2-1}^2 \subset G, \\ G_1 &\subset G_2^3 \subset G_3^3 \subset \dots \subset G_{k_3-1}^3 \subset G, \\ &\vdots \\ G_1 &\subset G_2^m \subset G_3^m \subset \dots \subset G_{k_m-1}^m \subset G. \end{aligned}$$

Then part of the diagram of lattice subgroup G is as shown in Figure 1.

According to Theorem 11, $n(F_{P_1=G_1}) = \sum_{i=2}^{k_1-1} n(F_{P_1=G_i^1}) + n(F_{P_1=G}) + \sum_{i=2}^{k_2-1} n(F_{P_1=G_i^2}) + \dots + \sum_{i=2}^{k_m-1} n(F_{P_1=G_i^m})$. Applying Theorem 12 and with a little algebraic manipulation, we have $n(F_{P_1=G_1}) = \sum_{i=1}^m 2^{k_i-2} - (m-1)$. $\square$

Corollary 13. *Let G be a group. Then*

$$n(F_{P_1=\{e\}}) = \sum_{\{e\} \neq H \leq G} n(F_{P_1=H}).$$

Proof. The proof follows immediately from Theorem 11. $\square$

Corollary 14. *Let G be a group. Then $n(F_G) = 2 \cdot n(F_{P_1=\{e\}})$.*

Proof. From (2), we have $n(F_G) = \sum_{H \leq G} n(P_1 = H)$. Therefore

$$n(F_G) = n(F_{P_1=\{e\}}) + \sum_{\{e\} \neq H \leq G} n(F_{P_1=H}).$$

Using Corollary 2, we have

$$n(F_G) = n(F_{P_1=\{e\}}) + n(F_{P_1=\{e\}}) = 2 \cdot n(F_{P_1=\{e\}}).$$

$\square$

Example 15. Consider a diagram of lattice subgroups of Z_2^3 (see Figure 1 in [7]). Using Theorem 8, we have

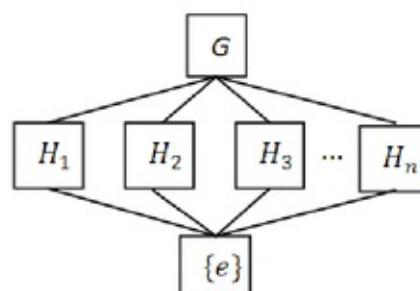
$$\begin{aligned} n(F_{P_1=H_{15}}) &= n(F_{P_1=H_8}) = n(F_{P_1=H_9}) \\ &= n(F_{P_1=H_{10}}) = n(F_{P_1=H_{11}}) = n(F_{P_1=H_{12}}) \\ &= n(F_{P_1=H_{13}}) = n(F_{P_1=H_{14}}) = 1. \end{aligned}$$

Applying Theorem 11 we have

$$\begin{aligned} n(F_{P_1=H_1}) &= n(F_{P_1=H_2}) = n(F_{P_1=H_3}) \\ &= n(F_{P_1=H_4}) = n(F_{P_1=H_5}) = n(F_{P_1=H_6}) \\ &= n(F_{P_1=H_7}) = 4. \end{aligned}$$

Therefore, using Theorem 11 we obtain $n(F_{P_1=H_0}) = 8(1) + 7(4) = 36$. Using Corollary 14 we determine the number of fuzzy subgroups of Z_2^3 to be 72.

Corollary 16. *Let $\{e\} \subset G_1 \subset G_2 \subset \dots \subset G_{k-1} \subset G_k = G$, be the only maximal chain from $\{e\}$ to G on the lattice subgroups of G . Then $n(F_G) = 2^k$.*

Figure 2: Diagram of lattice subgroups G .

Proof. The proof follows immediately from Corollary 14 and Theorem 8. $\square$

Example 17. Let p be a prime number. Then $\{0\} - Z_p - Z_{p^2} - Z_{p^3} - \dots - Z_{p^k}$ is the only maximal chain from $\{0\}$ to Z_{p^k} . Using Corollary 4 we obtain the number of fuzzy subgroups of Z_{p^k} to be 2^k .

Note that Theorem 3.1(1) in [4] is a special case of Corollary 16. It is asserted by the following theorem.

Theorem 18. (The number of fuzzy subgroups of p -group). Let G be a cyclic group of order p^k , where k is natural number and p is prime. Then $n(F_G) = 2^k$.

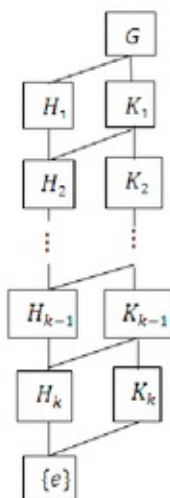
Proof. Let $G = \langle a \rangle$. The only maximal chain from $\{e\}$ to G is $\{e\} \subset \langle a^{p^{k-1}} \rangle \subset \langle a^{p^{k-2}} \rangle \subset \langle a^{p^{k-3}} \rangle \subset \dots \subset \langle a^{p^2} \rangle \subset \langle a^p \rangle \subset G$. Applying Corollary 16 directly, we have $n(F_G) = 2^k$. $\square$

Corollary 19. Let $\{H_1, H_2, \dots, H_k\}$ be the set of all nontrivial subgroups of G . If $\forall i, j \in \{1, 2, \dots, k\}$ with $i \neq j$ implies H_i is not subset of H_j and H_j is not subset of H_i , then the number of fuzzy subgroups of G is $2(k+1)$.

Proof. The diagram of lattice subgroups of G is given in Figure 2. According to Theorem 8 we obtain $n(F_{P_i} = H_i) = 1, \forall i = 1, 2, \dots, k$. From Theorem 8 and Theorem 11, we obtain $n(F_{P_1} = \{e\}) = 2(k) - (k-1) = k+1$. Using Corollary 14, we conclude that the number of fuzzy subgroups of G is $2(k+1)$. $\square$

Lemma 20. (Murali and Makamba, see Lemma 3.5 in [5]). For a prime p , the number of nontrivial subgroups of $G = Z_p \times Z_p$ is $p+1$.

Theorem 21. For a prime p , the number of fuzzy subgroups of $G = Z_p \times Z_p$ is $2p+4$.

Figure 3: Diagram of lattice subgroups G for $m=k$.

Proof. By Lemma 20, the number of nontrivial subgroups of G is $p+1$. All of them are cyclic of the order p . By Corollary 19, we obtain the number of fuzzy subgroups of $G = Z_p \times Z_p = 2p+4$. $\square$

Theorem 22. Let $\{H_1, H_2, \dots, H_m, K_1, K_2, \dots, K_m\}$ be the set of all nontrivial subgroups of G where $H_i \supset H_{i+1}, K_i \supset K_{i+1}, \forall i = 1, 2, \dots, m-1$ and H_i is not subset of K_i, K_i is not subset of $H_i, \forall i = 1, 2, \dots, m$. Then $n(F_G) = 2^m(m+2)$.

Proof. We prove by induction on m . The statement is true for the first $m = 1, 2$. Assume that the statement is true for $m = k$. This means that the number of fuzzy subgroups of G whose diagram lattice is shown in Figure 3 is $2^k(k+2)$. For $m = k+1$, the diagram lattice subgroups G is shown in Figure 4.

For the case where $m = k+1$, we have

$$n(F_G) = n(F_{P_1=\{e\}}) + n(F_{P_1=K_{k+1}}) + \sum_{i=1}^{k+1} n(F_{P_1=H_i}) + \sum_{j=1}^k n(F_{P_1=K_j}) + n(F_{P_1=G}).$$

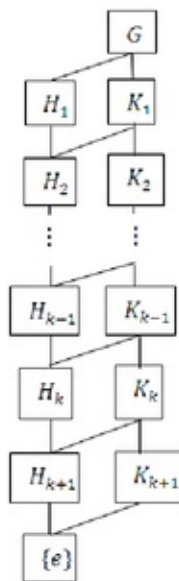


Figure 4: Diagram of lattice subgroups G for $m=k+1$.

Using Theorem 8, we have $n(F_{P_1=K_{k+1}}) = 2^k$. Thus, we have

$$\sum_{i=1}^{k+1} n(F_{P_1=H_i}) + \sum_{j=1}^k n(F_{P_1=K_j}) + n(F_{P_1=G}) = 2^k(k+2).$$

Hence, $n(F_G) = n(F_{P_1=\{e\}}) + 2^k + 2^k(k+2)$. Finally, using Corollary 13, we obtain $n(F_G) = 2[2^k + 2^k(k+2)] = 2^{k+1}(k+3)$. This complete the induction. $\square$

Corollary 23. (The number of fuzzy subgroups of $Z_p^n \times Z_q$) If p, q are distinct primes, then the number of fuzzy subgroups of $Z_p^n \times Z_q$ is $2^n(n+2)$.

Proof. Use Theorem 22. $\square$

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